LACUS FORUM XXXVII

Communication and Cognition: Multidisciplinary Perspectives

McMaster University

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LACUS FORUM XXXVII

COMMUNICATION AND COGNITION: MULTIDISCIPLINARY PERSPECTIVES

EDITED BY

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Metatheory

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VASTNESS REVISITED

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Abstract. In 1984 Terrence Langendoen and Paul Postal argued that the output of natural grammars, that is natural languages, had a degree of infinity that was larger than that of any set in mathematics. While their work seemed arcane and has not been widely taken up since, they did address a fundamental question that lies at the formal heart of generative grammar: what size of set does a grammar generate, or, how big is language?

Three years later McCawley criticized their work on the grounds that an infinitely long sentence was ungrammatical, his objection being a linguistic version of what in logic is termed the halting problem for a Turing machine. Langendoen and Postal could bypass McCawley's criticism by taking refuge in the distinction between a potential and an actual infinite. The nature of generative grammar would remain the same in either case.

In fact there is a hitherto unrecognized flaw in Langendoen and Postal's work that arises when they filter the output of their machine (a power set operation) that makes bigger sets from smaller ones. The resulting filtered set reverts back to a size, or cardinality, that is the same as the original set of infinitely long sentences, this cardinality being what in math is called denumerable, or in set theory aleph-null. Their effort to advance language to unbounded infinite heights stalls at the lowest level of infinity, aleph-null. Their hierarchy cannot be realized.

A simple and natural model of language use is put forward here: language is used in context to produce acceptable or unacceptable instances of a utterance – context pair. The context model is an extensional one that uses space-time as a natural base for an abstract space of speech. It can then be shown that the human ability to use language uses a generative capacity whose output is greater than aleph-null, that is, its cardinality is at least as large as that of the continuum (aleph-1) and probably that of all relationships on the continuum (aleph-2).

These findings form a natural generalization and extension of the Chomsky hierarchy to super-grammars. They also complete Chomsky's initial demands for generative grammar: an explanation not only for the infinite use of language, but also its spontaneous and creative use, the last two of which have to date only been partially met.

Keywords: vastness, recursion, generative paradigm, Chomsky hierarchy, performance, set theory, Turing test, Turing machines, degrees of infinity, super grammars.

Languages: (none)

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THE FORMAL STRUCTURE OF generative grammar may be represented simply as in (1):

(1) G(U) $\gamma\rightarrow$ {S} XOR {*S}

where G is the grammar that partitions, $\gamma\rightarrow$, or "generates," a set of utterances, U, into either acceptable sentences, {S}, or unacceptable ones, {*S}, by assigning derivations, Δ, and semantic readings, ρ, to members of the acceptable set, and XOR is boolean exclusive 'or.'

Some linguists may desire a system with degrees of grammaticality, thus complicating the picture in (1), but this demand could be accommodated as in (2a), where G assigns utterances to a graded series of sets, $\{S_n\}$, with a threshold of acceptability reached at some value of the index, let us say, k. This threshold can be used to partition the outcome again into two sets (of sets) (2b), duplicating the condition in (1).

(2) a. G $\gamma\rightarrow$ $\{S_n\}$

b. G $\gamma\rightarrow$ {S} XOR {*S}

where $\{S\}\supset\{S_i\}$, $1\leq i\leq k$, up to S_k acceptable, and $\{*S\}\supset\{S_j\}$, $k+1\leq j\leq n$ unacceptable; $\supset$ denotes 'contains as a subset.'

Crucially, **G** is finite, while both {**S**} and {***S**} are infinite. This is Chomsky's criterion for generative grammar, making infinite use of finite means (Chomsky 2006, 1975). Further, Chomsky sought to capture through this generative paradigm the infinite, spontaneous, and creative use of language. Iteration is assumed to explain the infinite nature of language, (the size of {**S**} in a sense that I shall make exact). I shall conclude by arguing otherwise, though I admit iteration and the denumerable nature of sentence structure. Iteration is not why language feels infinite. Nor does iteration or the formal schema in (1) in any way explain the spontaneous or creative use of language. One must seek the infinite feel of language in its use in context, that is in the performance of language, and it is there that spontaneity and creativity emerge.

1. CARDINALITY. Langendoen and Postal (1984) took up this fundamental feature of generative grammar as depicted in (1), and by doing so created a new field of linguistics, vastness theory. They generated a set of sentences, $\mathbf{S_0}$, with cardinality $\aleph_0$, (aleph-null, or aleph zero), the cardinality or size of the integers, these constituting a denumerably infinite set. They achieved this through the iteration of embeddings. To capture their effort symbolically, as in (3), let **E**, an embedding operator, act on a finite sentence, $\boldsymbol{\sigma}$, of length **k**, such that with $\mathbf{E^n(\sigma)} = \mathbf{\Sigma^n}$, $\boldsymbol{\sigma}$ embedded n-times. They then take the limit as $\mathbf{n}\rightarrow\infty$. Then their findings can be represented as in (3).

(3) a. $E^n(\sigma) = E(E(E\ldots(\sigma))\ldots) = \Sigma$, Σ "n times"

b. $\#(\mathbf{S_0}) = \#(\lim_{n\rightarrow\infty} E^n(\sigma)) = \#(Z) \times (n \times k) = \aleph_0 \times (n \times k) = \aleph_0$

c. $\#(\{\text{all of the words}\}) = \#(\lim_{n\rightarrow\infty}(n \times k) = \#(N) = \aleph_0$

where #() is the cardinality function that counts the size of a set, $\aleph_0$ is the cardinality of a denumerable infinity, the size of the set of natural (counting) numbers, **N**.

One should note that $\aleph_0$ is a transfinite cardinal, in fact, it is the least such cardinal, the lowest transfinite number. Transfinite cardinals are a generalization of the concept of size (Cantor 1955). When multiplying with a transfinite cardinal, as in (3b),

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(or with a series of transfinite cardinals), the biggest one always prevails (Devlin 1979: 82-8; Enderton 1977: 138-44; Kamke 1950: 17-51).

The actual sentence which Langendoen & Postal (1984:56-7) used was that in (4), simple but adequate for their formal purposes.

(4) S_0 = "I know that I know that … Babar is happy."

The infinity is in the middle of the sentence, so to speak. Their sentence has a beginning and an end, but an infinite middle, much like an infinite series approaching a limit as well as being defined by or equated to that limit. This choice enabled them to avoid the criticism that they had created a sentence without an end, and therefore one that was nongrammatical, though in fact such a criticism was leveled against them. The sentence in (4) consists of discrete words assembled into discrete clauses, and as such each word can be assigned an integer, starting with 1 at the beginning. In this way the size of the sentence, strictly speaking its cardinality, is in one-to-one correspondence with the integers. Hence the sentence has cardinality $\aleph_0$, which by definition is the same as that of the integers, **Z**.

2. REMARKS ON CARDINALITY. To avoid controversy, $\mathbf{S_0}$ must be viewed as a feature of competence, subject to a performance truncation (which can be denoted by ◇). In other words, one can never utter an infinite sentence, but it is in the nature of $\mathbf{S_0}$ that such a sentence could be assigned a derivation, that is, that it could be assigned to either {**S**} or {***S**}. One simply must stop because of performance constraints, that is, one must truncate one's sentence, ◇.

This problem of truncation is not unique to language. The same must be the case with the infinitude of the transcendental numbers. For example, π (the ratio of a diameter of a circle to its circumference), or e (the base of the natural logarithms) can never be represented, but must be understood. The symbols, 'π,' 'e', etc., are in fact limits. Truncation looks simple enough, but it formalizes a solution to the long-standing problem of real as opposed to potential infinities.

James McCawley noted (1987) that infinitely long sentences could be argued to be inherently ungrammatical. McCawley may have had in mind the halting problem for a Turing machine, the latter usually symbolized as $\mathbf{M_T}$. In modern experience this would be equivalent to an application freezing up on a computer, even if a well-defined end or goal exists. Langendoen and Postal deal with sentences that are locally grammatical, that is, any given phrase taken within a finite sampling window will be grammatical. One can then form an infinite union of all such locally grammatical substrings to produce a globally grammatical string, (5), even if it is unacceptable in a McCawley-Turing sense, that is, **G** or $\mathbf{M_T}$ fail to halt. Truncation also answers the objections raised against the initial work of Langendoen and Postal (as in Abbott 1986; Lapointe 1984; Rauf 1984; Sgall 1984; Thompson 1984).

(5) a. $G \rightarrow \sigma, \sigma \ \varepsilon \ \{S\}$ (local grammaticality)

b. $\cup_i(\sigma_i) = S_0 \mid S_0 \ \varepsilon \ \{S\}, 1 \leq i \leq \aleph_0$ (global grammaticality)

where ∪ is union or "joining", | is 'such that', and ε is 'an element of set.'

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With truncation, however, we can admit Chomsky's recursion into the structure of grammatical competence without running afoul of either performance constraints or Turing's halting criterion.

3. ASIDE ON SET THEORY.[1] For what I wish to prove, the concept of a power set, (6), is crucial. This is the set of all sets that can be formed from the elements of a given set. Among these sets are the null set, **ø**, and the whole set itself. It is as though one were to take a bag of fruit and choose all possible combinations of the fruit therein, including no choice, the null set, or the entire bag itself. Every choice leaves behind a remainder, which itself is a possible choice, so the choices come in pairs. This pairing of choices explains the general result in (8).

(6) a. A = {a, b, c}
b. Π(A) = {ø, {a}, {b}, {c}, {a, b}, {a, c}, {c, b}, {a, b, c}}
where Π, the power set operator, generates the set of all subsets, making bigger sets from smaller ones.

The size of **A**, its cardinality, is merely the number of elements contained within it, as in (7). Similarly the cardinality of **A**'s power set can be determined by simple counting, though now we can relate the two sizes, as in (8).

(7) #(A) = 3

(8) $\#(\Pi(A)) = 2^{\#(A)} = 2^3 = 8$

The relationship in (8) is can be generalized to all values of #(A).

Now, (8) may be extended to relationships such as (10), where we have assumed that the power set operator has once again faithfully created a set that is larger than its input.

(9) #(natural numbers) = #(N) = $\aleph_0$

(10) $\#(\Pi(N)) = \aleph_1 = 2^{\aleph_0}$

This is Georg Cantor's Continuum Hypothesis, usually abbreviated as CH, which is controversial (Cohen 1966). So, if one wishes, one may simply have (11).

(11) If #(A) = a, and #(Π(A)) = b, then $a < b$
(the power set is always bigger than its original set)

without committing to a hierarchy of alephs. In what follows I shall assume the CH. (The CH may be generalized. See (14) below.)

One may add a decimal (a set {.} of cardinality 1) to the natural numbers, **N**, (0, 1, 2, …). As with transfinite multiplication, the transfinite number prevails in addition (here denoted by +), so that $\boldsymbol{\aleph_0 + 1 = \aleph_0}$. One can then shuffle this new set of {**N** ∪ {**.**}} (a "shuffle product," see Eilenberg 1974:19). The result looks like the positive real numbers, $\mathbf{R}^+$, the numbers on the continuous line, such as 2.718281828… (***e***, the base of the natural logarithms), 3.141592653… (***π***), or 4.472135955… (the square root of 20). Cantor denoted the cardinality of the "reals" or of the continuum by $\boldsymbol{\aleph_1}$.

[1] See Enderton 1977, Kamke 1950, Devlin 1993, Kunen 1980, Kanamori 2000

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Many mathematicians find this seemingly sensible result controversial. Paul Cohen (1966) proved that the CH is independent of the other axioms of set theory, even though his famous forcing argument seems to assume the consequent. In other words, one can have set theory with or without the CH.

4. ASCENDING THE TRANS-FINITE LADDER.[2] Langendoen and Postal ambitiously argue that language climbs the transfinite ladder (Enderton 1977:9, Fig. 3). To do so, they form a power set of $\mathbf{S_0}$, $\mathbf{\Pi(S_0)}$. They then construct a new set, $\mathbf{S_1}$, putatively equinumerous (of the same cardinality) with $\mathbf{\Pi(S_0)}$. This new set consists of sentences from $\mathbf{\Pi(S_0)}$ rendered grammatical by means of conjunction, since the elements of $\mathbf{\Pi(S_0)}$, being scrambled from $\mathbf{S_0}$, are ungrammatical:

(12) $\Pi(S_0)$ = {[I know [that B…]], *[[that B is happy] I know [that B is happy]], …}

(13) S_1 = {[I know [that B…]], [[I know [that B…] and [I know [that I know [that B…]]]], …]B…]]]], …]

They assume the CH. (Once the CH is assumed, the generalized CH, or GCH, may be employed and the set theory will remain consistent.) They then assert that $\mathbf{S_1}$, as with $\mathbf{\Pi(S_0)}$, has the power of the continuum, $\aleph_1$. This is only their first step with the power set operator. They then form the power set of $\mathbf{S_1}$ and proceed ad infinitum. In this way they claim to ascend the transfinite ladder, by means of the generalized continuum hypothesis, as in (14).

(14) $\aleph_{n+1} = 2^{\aleph_n}$

They conclude that language has truly extraordinary cardinality, beyond the size of any set.

5. STUCK ON NULL, $\aleph_0$. $\mathbf{\Pi(S_0)}$ is not well ordered lexicographically (Kunen 1980: 173-75). To proceed up the transfinite hierarchy and retain grammaticality, Langendoen and Postal must clean up the output of their power set operation. $\mathbf{S_1}$ is supposed to salvage grammaticality from the chaos of $\mathbf{\Pi(S_0)}$, but their way of forming $\mathbf{S_1}$ from $\mathbf{\Pi(S_0)}$ can be shown to limit the cardinality of $\mathbf{S_1}$ to $\aleph_0$. Any further iteration in the manner they envisage fails to create sets of higher cardinalities because of the necessity of each time filtering out ungrammatical forms generated through $\mathbf{\Pi(S_n)}$. In effect their formation of $\mathbf{S_1}$ can be shown to be achieved through a denumerable application of conjunction, **&**, which may be viewed as an operator acting $\aleph_0$ times simply on $\mathbf{S_0}$, so that each element of $\mathbf{S_1}$ in (13) consists of a conjunction of finite sentences. The whole is therefore composed of an infinite conjunction of these elements. If one allows the conjoined sentences themselves to proceed toward an infinite limit, then at most one has:

(15) $\#(\&(S_0)) = \aleph_0 \times \aleph_0 = \aleph_0 = \#(S_0)$

No further ascent into the transfinite beyond is possible, since any power set must be filtered for grammaticality through the conjunction operator.

[2] See Enderton 1977:7-9.

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6. Super grammars (Turing test grammars). while the cardinality of the sets in (1) remains denumerable, this technicality does not directly shape our sense of language's infinitude (see the suggested extension in Uriagereka 2008). I take a different approach, one based on performance, and show that the sense of language's infinity is instead captured by appropriate use, that is by taking language in context, with no final use being self-evident.

I posit an event, **Ω**, consisting of two elements:

(16) $\Omega = (C, U)$, with C = context and U = utterance

Appropriate use means to parse the set {(**C**, **U**)} into acceptable {(**C**, **U**)} and unacceptable {*(**C**, **U**)} subsets by the generative effects of a super-grammar, Γ. Γ is equivalent to the Turing test for artificial intelligence, wherein one may speak to a machine without being able to distinguish it from a human. Hence I am tempted to use an alternate designation, "Turing Test Grammar." The process in (17) is superficially analogous to that in (1).

(17) $\Gamma(\Omega)\ \gamma\!\rightarrow \{(C, U)\}$ XOR $\{*(C, U)\}$

A determination of the cardinality of the sets in (17) will show that Γ is fundamentally different from G.

7. The cardinality of {(c, u)}. Generative capacity is a measure that depends both on the Kolmogorov complexity (Li and Vitányi 2008) of a grammar's output (complexity of the derivations or structures assigned) and upon the cardinality of that output. For shorthand I shall simply assign an extended cardinality to a grammar, (18) and (19), equal to that of its output(s), though strictly speaking it should represent the number of processes or rules and the size of the lexicon contained in a given **G** and accordingly be a finite, in fact, relatively small number. In (18) I take **G** to be $\mathbf{G_0}$, the most complex grammar of the Chomsky hierarchy, though each grammar of the hierarchy has the same cardinality.

(18) $\#(G_0) = \#(L_0) = \#(\{S\}) = \aleph_0$

For the super-grammar in (19), its extended cardinality is that of its super-language, **Λ** which is the product of the components of **Λ**, namely the context and the utterance.

(19) $\#(\Gamma(\Omega)) = \#(\Lambda) = \#(\{(C, U)\}) = \#(\{C\}) \times \#(\{U\})$

Whatever grammatical paradigm is used to model {U}, its cardinality is plainly that of L_0, as in (20).

(20) $\#(\{U\}) = \aleph_0$

Clearly the answer to (19) depends upon a reasonable model of context.

I will take {**C**} to be set of all contexts in life, which has the world as its stage. More specifically contexts take place in space-time, **Ξ**. In Figure 1, I depict the world

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lines, as four dimensional representations are called, of two people, let us call them Billy Bob and Peggy Sue, as they move through space and progress through time.

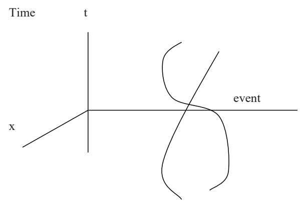

Figure 1: World-lines in Space-time, Ξ, (two people)

Such world lines, however, depict the mere physical aspects of Billy Bob and Peggy Sue. While contexts are certainly set in **Ξ**, they also are constructed from cultural perceptions and conceptions. Significant events or states are functions of these space-time contexts. I shall term a significant event or state a "history," **H**. One might view a history, **H**, as a function of individuals, egos, in a context, **C**, as in (**21**).

(21) H = *f*({ego}, C)

The cardinality of {ego} is finite, since there are only so many people in history.

World-lines are continuous, but the elements of **H** can be discontinuous. In fact histories begin, end, intersect, and avoid one another, all of which conditions can be highly significant for the parties concerned. In Figure 2 I have drawn (crudely) the legal context of Billy Bob and Peggy Sue's marriage. The bottom bar represents the moment when Billy Bob and Peggy Sue are married in the company of family, followed by a brief honeymoon, (the unified stretch), and then by more normal, independent married life, all of which has legal significance. Their prior, independent lives are not relevant to legal considerations and therefore are not a part of the history depicted in Figure 2.

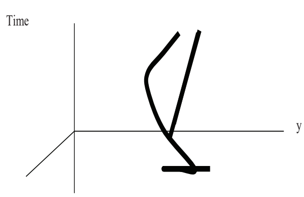

Figure 2: History in Space-time (the two are married)

The set of all histories, {**H**} = $\mathcal{H}$, is made of subsets of **Ξ**, and combinations of subsets of **Ξ**. To achieve $\mathcal{H}$ one must take the power set of **Ξ**.

(22) $\mathcal{H} = \Pi(\Xi)$

It is important when forming a power set of a continuum that it be done so as to permit non-continuous or non-connected forms, otherwise a mere diffeomorphism (smooth distortion) results and the cardinality is not increased (as in Uriagereka 2008;

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Postal 2004). (22) achieves this because of the conditions on histories. Technically, space-time Ξ serves as the base space for the fiber bundle of histories, $\mathcal{H}$, a separate cognitive space imposed upon space-time (Morita 2001:232; Frankel 1997:ch. 17). We may now calculate the cardinality of {**C**}.

Assuming the GCH in (23) – (25), the cardinality of space-time Ξ is:

(23) $\#(\Xi) = \aleph_1$

That of all possible histories, (21), these being functions of individuals in contexts set within Ξ, is:

(24) $\#(\mathcal{H}) = \#(\Pi(\Xi)) = 2^{\aleph_1} = \aleph_2$

The result in (24) implies that the cardinality of $\mathbf{\Lambda}$, that is, of $\{(\mathbf{C}, \mathbf{U})\}$ is:

(25) a. $\#(\mathcal{H}) = \#(\{f(\{\text{ego}\}, C)\}) = \#(\{\text{ego}\}) \times \#(\{C\}) = \#(\{C\}) = \aleph_2$
b. $\#(\Lambda) = \#(\{(C, U)\}) = \#(\{C\}) \times \#(\{U\}) = \aleph_2 \times \aleph_0 = \aleph_2$

The result in (25) implies that our ability to use language spontaneously and creatively is beyond the power of any deterministic algorithm or a $\mathbf{G_0}$ ($\mathbf{M_T}$), since these produce only denumerable sets, $\aleph_0$.

Therefore the extended cardinality of $\mathbf{\Gamma}$ is $\aleph_2$. This is equivalent to the set of all possible functions and relations on the continuum. Such a system would seem spontaneous and creative, precisely because no deterministic grammar can achieve this level; nor can one achieve $\aleph_1$. An algorithm with a random application might achieve the power of the continuum in capacity if not execution, $\aleph_1$. One might now reasonably ask what conceivable sort of machinery lies within $\mathbf{\Gamma}$, that is, what can attain $\aleph_2$.

8. Representations of a super grammar. a super grammar matches contexts with utterances, so one might treat these as an "inner product," written **<…|…>**, of infinitely dimensional vectors, that is as a Hilbert Space (Young 1988):

(26) $\Gamma(\Omega) = \{\langle C|U\rangle\}$

One could then assign an acceptability measure to these inner products, with a threshold of acceptability, much as in (27):

(27) $\Gamma_\mu(\Omega) \rightarrow \mu(\langle C|U\rangle)$, if $\mu < \Theta$, threshold ,then $(C, U) \in \{*(C, U)\}$

In other words if the usefulness of the utterance in a given context falls below a threshold of appropriateness, $\mathbf{\Theta}$, with $\mathbf{\Theta}$ being determined empirically, socially, or logically, then the utterance is not acceptable or the context has been misconstrued by the speaker.

Symmetries within {**U**}, that is, grammatical constraints, and symmetries within {**C**}, perceptual and cultural constraints, would then dictate the outcome of $\mathbf{\Gamma}$ under the acceptability measure. Such constraints have been the subject of generative grammar for several decades now, while similar perceptual and cultural constraints have been the object of study by cognitive scientists and some anthropologists. The effective study of super-grammars would require a new degree of collaboration among these three parties.

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9. SPECULATIONS. Work by Gödel on the hierarchy of infinite languages (Kanamori 2007) suggested that such super-grammars might, as a formal class, be subject to extension up the scale of transfinite numbers, as Langendoen and Postal once hoped for $\mathbf{G_0}$. If we equate or rename $\mathbf{G_0}$ as $\mathbf{\Gamma_0}$, then we might extend and generalize the Chomsky hierarchy as in (28).

(28) $G_0 = \Gamma_0 < \Gamma_1 < \Gamma_2 < \Gamma_3 < \ldots < \Gamma_n < \ldots$

$\mathbf{\Gamma_1}$ might be our ability (and that of "higher" animals) to cope with space-time in a thinking fashion, while $\mathbf{\Gamma_2}$ would be our ability to make reflective sense of our lives, to formulate histories in contexts. Any higher super-grammar would seem to lie beyond the current level of the human mind. And above would be features of a genuine super-mind.

The inner product in (26) bears a superficial resemblance to the Dirac notation used in quantum mechanics (for example, Liboff 1980:93-99). If taken seriously, this model suggests an anti-Cartesian paradigm, namely, that the universe is more like a mind than it is like a clock-work (see too Conway and Kochen 2009) . Randomness is an inherent feature in both. Without randomness, or frozen accidents (Hartle 2003:46, quoting Murray Gell-Mann) in mind and the physical world, existence would have no informational content. All would be predictable and therefore lack information, as the definition of information in (29), when applied to certainty, yields zero. Here **P** is the probability of an event occurring. Since this ranges from zero (cannot happen) to one (certainty), **P** is normally negative. The minus sign therefore makes the information, **I**, a positive number.

(29) $I = -\log_2(P) = -\log_2(1) = 0$

A careful consideration of what spontaneous and creative use of language requires has led us to some remarkable conclusions about grammar and the mind. Further work along the lines adumbrated here promises to enrich our view of both ourselves and of our world.

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REFERENCES

ABBOTT, BARBARA. 1986. *Review of Langendoen & Postal.* 1984. Language 62: 154-57.

CANTOR, GEORG. 1955. *Contributions to the founding of the theory of numbers.* Trans. by Philip E. Cantor. Reprint of the 1915 Open Court edition. New York: Dover.

CHOMSKY, NOAM. 1975. *Reflections on language, New York: Pantheon Books.*

—-. 2006. *Language and mind (third edition).* Cambridge, UK: Cambridge University Press

COHEN, PAUL J. 1966. *Set theory and the continuum hypothesis.* Reading, MA: Benjamin/Cummings.

CONWAY, JOHN H AND SIMON KOCHEN. 2009. *The Strong Free Will Theorem.* Notices of the AMS [American Mathematical Society], volume 56, no. 2, pp. 226-232.

DEVLIN, KEITH. 1979. *The joy of sets.* (2nd edition 1993.) Berlin: Springer Verlag

EILENBERG, SAMUEL. 1974. *Automata, languages, and machines, volume A.* New York: Academic Press.

ENDERTON, HERBERT B. 1977. *Elements of set theory.* New York: Academic Press.

FRANKEL, THEODORE. 1997. *The geometry of physics, an introduction.* Cambridge, UK: Cambridge University Press.

HARTLE, JAMES. 2003. *Theories of everything and Hawking's wave function of the universe.* In The future of theoretical physics and cosmology, ed. by G. W. Gibbons, E. P. S. Shellard, and S. J. Rankin, 38-50. Cambridge, UK: Cambridge University Press.

KAMKE, E. 1950. *Theory of sets.* Trans. by Frederick Bagemihl. New York: Dover.

KANAMORI, AKIHIRO. 2000. *The higher infinite.* Berlin: Springer Verlag.

—-. 2007. *Gödel and set theory.* The bulletin of symbolic logic 13:153-88.

KUNEN, KENNETH. 1980. *Set theory.* Dordrecht: Elsevier.

LANGENDOEN, D. TERRENCE, & PAUL M. POSTAL. 1984. *The vastness of natural languages.* London, UK: Blackwell.

LAPOINTE, STEVEN G. 1986. *Review of Langendoen & Postal.* 1984, Linguistics and philosophy 9.225-43.

LI, MING, & PAUL VITÁNYI. 2008. *An introduction to Kolmogorov complexity and its applications.* 3rd edition. Berlin: Springer Verlag.

LIBOFF, RICHARD L. 1980. *Introductory quantum mechanics.* Oakland, CA: Holden-Day, Inc.

MCCAWLEY, JAMES D. 1987. *Review of Langendoen & Postal 1984, International journal of American linguistics 53:236-39.*

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MORITA, SHIGEYUKI. 2001. *Geometry of differential forms.* Trans. by T. Nagase and K. Nomizu. Providence Rhode Island: American Mathematical Society.

POSTAL, PAUL M. 2004. *Skeptical linguistic essays.* Oxford, UK: Oxford.

RAUFF, JAMES V. 1989. *Review of Langendoen & Postal.* 1984, Computational linguistics 15: 55-7.

SGALL, PETER. 1987. *Review of Langendoen & Postal 1984, Prague bulletin of mathematical linguistics 47: 63-68.*

THOMPSON, HENRY. 1986. *Review of Langendoen & Postal 1984, Journal of linguistics 22: 241-2.*

URIAGEREKA, JUAN. 2008. *An adjunct space.* Unpublished MS, University of Maryland.

YOUNG, NICHOLAS. 1988. *An introduction to Hilbert space.* Cambridge, UK: Cambridge University Press.

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